ФІНАНСОВІ ІНСТРУМЕНТИ РЕГУЛЮВАННЯ ІНВЕСТИЦІЙНОЇ АКТИВНОСТІ

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CALIBRATION OF SVI MODEL

The SVI implied volatility model is a parametric model for stochastic implied volatility. The SVI parameterization of the implied volatility smile was originally devised at Merrill Lynch in 1999. The popularity of the model is supported by its 2 properties:

1) The model satisfies Lee's Moment Formula which specifies the asymptotics for implied volatility. It means that for a fixed time to maturity t, the implied Black-Scholes variance $\sigma_{BS}^2\,(k,t)$ is linear in the log-strike k;

2) SVI model is easy to calibrate to market data so that the corresponding implied volatility surface is free of calendar spread arbitrage.

The aim of the paper is theoretically develop parameterization of the SVI model such that model should be able to detect static arbitrage and eliminate it by a re-calibration. For this, I have to ensure that calibration method gives no calendar spread arbitrage and butterfly arbitrage. Then I describe calibration method to fit the SVI model to market data. Finally, I present numerical results of the calibration method on the S&P500 index.

First of all, I need to provide notations I use in the project. S_t is a stock price process with natural filtration F_t. Forward price of European option is defined as $F_t{:=}E[S_t|F_t]$. $C_{BS}(k,\sigma^2 t)$ is Black-Scholes price of a European Call option. $\sigma_{BS}(k,t)$ is Black-Scholes implied volatility, andtotal implied variance is $\omega(k,t) = \sigma_{BS}^2\,(k,t)t$. The implied variance v is calculated as $v(k,t) = \sqrt{\ }\sigma_{BS}^2\,(k,t) = \omega(k,t)/t$. $\theta_{imp} = t\sigma_{BS}(k,t)$ is a total implied volatility. K is the strike price, k is log strike price, and x is a moneyless of the European Call Option.

Next, the SVI parameterization of the implied volatility must eliminate static arbitrage. Here I define what static arbitrage is. A static arbitrage opportunity is a dynamic arbitrage opportunity where positions in the underlying at a particular time only can depend on time and actual corresponding price. On the other hand, a dynamic arbitrage opportunity is a costless trading strategy that gives a positive future profit with positive probability and has no probability of a loss. The main aim of SVI model is to develop volatility surface that is free of static arbitrage:

1) it is free of calendar spread arbitrage;

2) each time slice is free of butterfly arbitrage. The slice of the volatility surface can be described as $k \to \omega(k,t)$

One way to force the SVI implied volatility surface to be static arbitrage risk-free is to use Kellerer's theorem. The theorem, with respect to European Call Option prices, states that for a given process of the Call Option price (integrable stochastic process) there exists a martingale $Z_t = Z_0 + \int_0^t \sigma(s, Z_s)dB_s$, where $\sigma(t,k) = \sqrt{\frac{2\partial C}{p\partial t}}$ and $p = \frac{\partial^2 C}{\partial x^2}(t,x)$. Until the process $Z_t$satisfies Lipschitz-Markov property and has a unique and strong solution, then we have static arbitrage free surface of implied volatility of a European Call Option.

Applying sufficient conditions of Kellerer's theorem for ensuring that the implied volatility surface of the European Call Option is free of static arbitrage, the next conditions should hold:

1) $\frac{\partial C}{\partial t}$ >0, where $t$is time to maturity;

2) $\lim_{K\to\infty} C(t,K) = 0$, where K is strike price;

3) $\lim_{K\to\infty} C(t,K) + K = a$, where $a \in R$;

4) $C(t,K)$ is convex in K;

5) $C(t,K)$ is non-negative.

All these conditions are from Kellerer's theorem. The conditions 1)-5) on European Call.

Prices are implied by the following conditions on the implied volatility surface:

a) $\frac{\partial \omega_{imp}}{\partial t} = \frac{\partial \theta_{imp}^2}{\partial t}$;

b) $\lim_{K\to\infty} d_1 = -\infty$, where d_1 is from BS formula;

c) $\theta_{imp} \geq 0$;

d) $(1 - \frac{x}{\theta_{imp}} \frac{\partial \theta_{imp}}{\partial x})^2 - \frac{\theta_{imp}^2}{4}(\frac{\partial \theta_{imp}}{\partial x})^2 + \theta_{imp} \frac{\partial^2 \theta_{imp}}{\partial x^2}$ ≥0. This condition can also be referred to as Durrleman's condition.

These conditions are necessary to ensure that implied volatility surface is free of calendar spread arbitrage and butterfly arbitrage. Calendar spread arbitrage is usually expressed as the monotonicity of European call option prices with respect to maturity. Thus, a volatility surface ω is free of calendar spread arbitrage if $\partial\omega/\partial t(k,t) \geq 0$, for all $k \in R$ and t >0.This condition is sufficient and necessary to ensure that implied volatility parameterization is free of calendar spread arbitrage.

Absence of butterfly arbitrage corresponds to the existence of a risk-neutral martingale measure and the classical definition of no static arbitrage. A slice is said to be free of butterfly arbitrage if the corresponding density is non-negative. For this, I use BS formula for a European Call Option. I define function g(k) as a density function of log strike prices of the European Call Option:

$$g(k) := \left(1 - \frac{k\omega'(k)}{2\omega(k)}\right)^2 - \frac{\omega'(k)^2}{4}\left(\frac{1}{\omega(k)} + \frac{1}{4}\right) + \frac{\omega''(k)}{2}$$

A slice is free of butterfly arbitrage only if $g(k) \geq 0$ for all $k \in R$ and $\lim_{k\to\infty} d_+(k) = -\infty$.

In conclusion, in this section I elaborated by means of Kellerer's theorem necessary conditions that ensure that implied volatility surface of the parametrized SVI model is static arbitrage-free.

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FINANCIAL INSTRUMENT OF ADJUSTMENT OF THE INVESTMENT PROCESS

The current post-crisis state of the Ukrainian economy, the emergence of the need for significant investments, unprofitable in terms of private capital, but necessary to continue reproduction on a national scale, and the failure of the «free market» to address these problems, require state intervention in the process of investment regulation. On the other hand, the course of political processes, the presence of the so-called «shadow» sector of the economy and, at the same time, the absence of clear mechanisms for the implementation of the chosen investment strategy negatively affect the investment climate and, in particular, the international investment image of Ukraine, which causes both losses of potential investment resources and outflow available capital.