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THE MAIN TRANSPORT PROBLEMS

The transportation problem is a classic optimization problem in operations research, where the objective is to minimize the total transportation cost of goods from several sources to several destinations. The problem involves determining the optimal shipping quantities between the sources and destinations, subject to supply and demand constraints [1].

The transportation problem can be formulated as a linear programming model, where the objective is to minimize the total transportation cost. Let us consider m sources and n destinations, with the supply and demand for each source and destination given by a_i and b_j, respectively. Let x_{ij} be the amount shipped from source *i* to destination *j*. Then the transportation problem can be formulated as follows:

$$Z(X)=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}\to Min \tag{1}$$

$$\sum_{j=1}^{n}x_{ij}=a_i,\ i=1,2,...m \tag{2}$$

$$\sum_{i=1}^{m}x_{ij}=b_j,\ j=1,2,...n \tag{3}$$

$$x_{ij}\geq 0,\ i=1,2,...m,\ j=1,2,...n. \tag{4}$$

where c_{ij} is the unit transportation cost from source *i* to destination *j*.

The transportation problem can be solved using various methods, including the simplex method, the network simplex method, and the transportation simplex method. The transportation simplex method is a specialized version of the simplex method that exploits the problem's special structure to solve it more efficiently.

The transportation simplex method involves constructing an initial feasible solution and iteratively improving it until an optimal solution is reached. The initial

feasible solution can be obtained using various methods, such as the northwest corner rule, the least cost rule, and the Vogel's approximation method.

The basic idea of the transportation simplex method is to identify a non-basic variable with a negative reduced cost and enter it into the basis. Then, a basic variable with the smallest ratio of the right-hand side value to the entering variable coefficient is selected to leave the basis. This process is repeated until an optimal solution is reached.

The transportation problem is a fundamental problem in operations research, with various practical applications in logistics, supply chain management, and transportation planning. The problem can be formulated as a linear programming model and solved using various methods, including the transportation simplex method. The transportation simplex method is an efficient method that exploits the problem's special structure to solve it more efficiently.

There are three main types of transportation problems:

1. Balanced Transportation Problem: In this type of problem, the total supply from all sources is equal to the total demand at all destinations. This means that the problem has a feasible solution.

2. Unbalanced Transportation Problem: In this type of problem, the total supply from all sources is not equal to the total demand at all destinations. This means that the problem does not have a feasible solution, and some additional constraints or adjustments are required to make the problem feasible.

3. Degenerate Transportation Problem: In this type of problem, one or more of the allocation variables take on a value of zero in the optimal solution, leading to a degenerate solution. This can cause problems with some solution methods, such as the basic transportation simplex method, and requires special handling techniques.

The solution methods for the different types of transportation problems are as follows:

The balanced transportation problem has an equal total supply and demand, and it can be solved using the transportation simplex method. The method involves finding an initial feasible solution, identifying the basic variables and calculating

their costs, and then iteratively improving the solution until an optimal solution is reached.

The unbalanced transportation problem has an unequal total supply and demand, and it requires additional constraints or adjustments to make it feasible. There are several methods for solving unbalanced transportation problems, such as:

- Adding a dummy source or destination to balance the problem
- Reducing the supply or demand to balance the problem
- Using the stepping-stone method to find an optimal solution.

The degenerate transportation problem has one or more allocation variables that take on a value of zero in the optimal solution. This can cause problems with some solution methods, such as the basic transportation simplex method. To solve a degenerate transportation problem, the following methods can be used:

- The modified transportation simplex method, which avoids cycling and ensures convergence to the optimal solution
- The minimum cell cost adjustment method, which modifies the problem to eliminate degeneracy
- The Russell's method, which modifies the problem to avoid degeneracy.

In summary, the transportation problem can be solved using various methods depending on the type of problem. The transportation simplex method is used for balanced transportation problems, while unbalanced and degenerate transportation problems require additional techniques such as adjusting the problem, using the stepping-stone method, or using modified solution methods to obtain optimal solutions.

REFERENCES

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