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APPLYING OPTIMIZATION THEORY TO BUILDING FUZZY VORONOI DIAGRAMS

The paper demonstrates the possibility of constructing fuzzy Voronoi diagrams based on a single approach: the formulation of a continuous problem of optimal partitioning of sets from n-dimensional Euclidean space into subsets with a quality criterion that provides the appropriate type of Voronoi diagram.

The theory of optimal partitioning of sets is a universal mathematical approach for constructing Voronoi diagrams, which is based on the following general idea. The initial problems of optimal partitioning of sets, which are mathematically formulated as infinite-dimensional optimization problems, are reduced through the Lagrange functional to auxiliary finite-dimensional non-smooth maximization problems or non-smooth maximin problems, for the numerical solution of which modern effective optimization methods are used. The peculiarity of this approach is the fact that the solution of the initial infinite-dimensional optimization problems can be obtained analytically in an explicit form, and the analytical expression can include parameters that are sought as the optimal solution of the above-mentioned auxiliary finite-dimensional optimization problems with non-smooth objective functions.

The universality of the proposed approach to the construction of Voronoi diagrams is also confirmed by the fact that the models and methods of solving continuous problems of optimal partitioning of sets can be generalized to the case of a vague assignment of the initial parameters of the problem or the requirement of a vague partitioning of the set, as a result of which the resulting Voronoi diagrams can have a vague nature.

The paper proposes an algorithm for constructing one of the variants of fuzzy Voronoi diagrams, when the set of points forming a Voronoi cell can be fuzzy.

The algorithm was developed based on the synthesis of the methods of the theory of optimal partitioning of sets and the theory of fuzzy sets.

The developed algorithm is programmatically implemented in C# on the WPF platform and using the 3D Unity Engine. To speed up calculations, the technology of calculations on a graphics accelerator (GPGPU) using Compute Shaders in the HLSL language was used. This turned out to be possible and advantageous from the point of view of the algorithm's running time, since the algorithm for calculating the fuzzy partition needs to calculate the value of the membership function at all nodes of the grid at each iteration, while these calculations are independent of each other and can be calculated in any order, i.e. are ideal for parallel computing. At the same time, since the problem of fuzzy partitioning with fixed centers is an inner problem for the problem of fuzzy partitioning with placement, that is, it is performed at each iteration, the acceleration of its calculations is justified. In fact, compared to calculations on the processor, the working time is ten times different, and it allows building images of partitions almost in real time. With the help of the developed software, the work of the algorithm is tested on examples of building standard, additively and multiplicatively weighted diagrams with fuzzy Voronoi cells.

The result of applying the theory of continuous problems of optimal partitioning of sets as a universal mathematical approach for constructing a Voronoi diagram and its generalizations is the possibility to construct not only already known Voronoi diagrams, but also to construct new ones.

REFERENCES

3. Kiseleva O. M. The Emergence and Formation of the Theory of Optimal Set Partitioning for Sets of the Euclidean Space. *Journal of Automation and Information Sciences*. Volume 50. 2018. p. 1-24.DOI: 10.1615/JAutomatInfScien.v50.i9.10
4. Zimmerman H.J. Fuzzy Sets Theory and Its Applications. 4th ed. Boston: Kluwer Academic Publishers. 2001. 514 p.