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INVERSE PROBLEMS OF REPRODUCING OF PHYSICAL AND MECHANICAL FIELDS IN INACCESSIBLE REGIONS

Inverse problems are those problems in which the cause must be calculated from the given effect. For example, based on the values of the acceleration of the body, you need to calculate the force acting on the body. Another example: based on the given values of the force acting on the body, it is necessary to calculate the force field (gravitational, electromagnetic). Inverse problems include problems determination of some physical properties of objects, such as density, coefficient of thermal conductivity, modulus of elasticity depending on coordinates or in the form of functions of other parameters. The success of the procedure for solving such problems depends to a large extent on the quality and quantity of the information received information experiment, as well as from the method of its processing. Let us consider the problem of restoration of the potential field in the inaccessible region based on the given values of measurements of this field in the accessible region.

Let some physical field be described by the Laplace equation:

$$\Delta u = 0 \qquad (1)$$

The area of existence of the field Ω consists of two parts: $\Omega = \Omega_1 \cup \Omega_2$. The part Ω_1 is available for measuring field characteristics. Measurement in the area Ω_2 is impossible. Let the points $P_1, P_2, \dots, P_n \in \Omega_1$ have known (measured) potential values $u_1, u_2, \dots, u_n$. It is necessary to construct a function $u^*(x)$, that satisfies equation (1) at the points of the entire region Ω, and at the measurement points $P_1, P_2, \dots, P_n \in \Omega_1$ will be close to the measured values $u_1, u_2, \dots, u_n$. In real applied problems, the points are located along the perimeter of some flat figure (in the case of a two-dimensional problem) or on the boundary of some body (in the case of a three-dimensional problem).

Let $\varphi_1(x), \varphi_2(x), \dots, \varphi_m(x)$- be linearly independent solutions of the Laplace equation. Then their linear combination

$$v_m(x) = \sum_{i=1}^{m} A_i \, \varphi_i(x) \qquad (2)$$

will also be a solution to this equation. As functions $\varphi_i(x)$ you can take harmonic polynomials or functions of the form $G(x-\beta_i)$, where $G(x)$ is the fundamental solution (Green's function) of the Laplace operator, β_i is a set of points (location of sources) that must lie outside the region Ω. The location of these points can be chosen arbitrarily, but for better accuracy it is recommended to choose their location uniformly aroundΩ_2. Let's find the coefficients $A_1, A_2, \ldots, A_m$ from the condition of proximity $v_m(x)$ to the measured potential values at the points $P_1, \ldots, P_n \in \Omega_1$. The standard deviation is used as a measure of proximity:

$$F(A_1, A_2, \ldots, A_m) = \sum_{j=1}^{n}\left[\sum_{i=1}^{m} A_i\,\varphi_i(x_j) - u_j\right]^2. \quad (3)$$

The coefficients of the linear combination (2) are found from the condition of the minimum of the function $F(A_1, A_2, \ldots, A_m)$. This condition gives m linear algebraic equations. For large values of m, the problem of finding the minimum of function (3) is ill-posed, as the system of linear equations is unstable. To construct an approximate solution of the obtained system of linear equations, we apply the Tikhonov regularization method [1–3].

The idea of the method is to replace the incorrect extremal problem for the initial functional *J(v)* with a sequence of correct extremal problems. Most often, during the implementation of this idea, the so-called Tikhonov functionals are used:

$$T(\mathrm{v},\alpha)=J(v)+\alpha\Omega(v), \quad (4)$$

where *J(v)* – is the functional of the initial problem. By *Ω(v)* we mean some functional defined on a subset of the domain of the initial functional, which acquires only non-negative values. The numerical parameter $\alpha \geq 0$ is called the regularization parameter; at α=0 the functionals *T(v, a)* and *J(v)* coincide.

Model examples consider the qualitative dependence of the solution on the value of the regularization parameter. The solutions of the system of linear algebraic equations should be those for which the error due to regularization is comparable to the measurement error and does not have critical deviations due to the instability of the system.

In further studies, it is planned to investigate in more detail the influence of the choice of location of source points β_i on the accuracy of calculations. Another interesting problem that is planned to be investigated is the construction of a basis for the case of discontinuous boundary conditions. After all, in places of sharp field difference, this method is not applicable in its usual form due to the presence of the Gibbs phenomenon.

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MODELING OF GENERATIVE RECOMMENDATION SYSTEMS

The goal of this work is to describe shortly the approaches to building the system which provide recommendations in the form of an ordered sequence of HTML pages of the resource that is proposed to the user [1; 2; 4; 5] and the results, obtained in the process of research. To perform the ranking procedure, which is in the center of different investigations [4; 6; 7], statistical information about user transitions between web pages is used. The web resource model is depicted as a web graph as well as the user behavior model is represented as a graph of transitions between resource pages. In its turn, the web graph is represented by an adjacency matrix, and the transition graph has a weighted probability matrix of transitions between vertices.

It is considered that user transitions between web resource pages can occur by entering a URL in the browser's address bar or by following a link on the current page [3]. User transitions between vertices in a finite graph, in accordance with probabilities determined by the weight of the graph edges, are represented by a homogeneous Markov chain and are considered as a random walk process on the graph with the possibility of transitioning to a random vertex. It is noteworthy